Sec 1.3 slope fields and solution curves.

An ODE $\frac{dy}{dx} = f(x,y)$ specifies a slope (L to R) at each point (a,b). ("slope field")

Solution curves are the curves traced by the graphs of solutions of the ODE

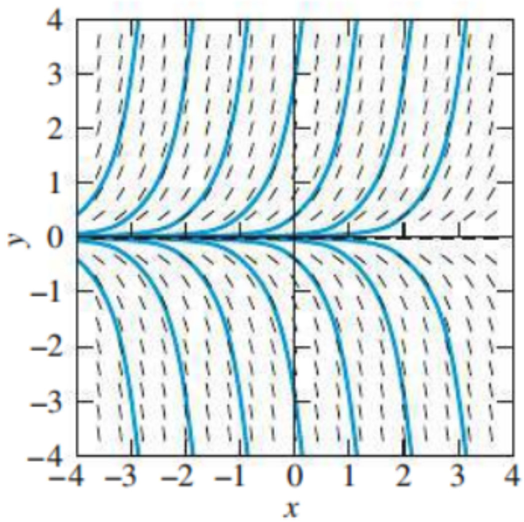


FIGURE 1.3.2(a) Slope field and solution curves for $y' = 2y$.

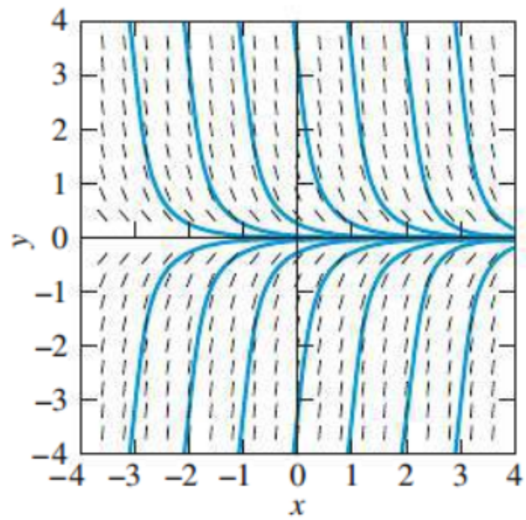
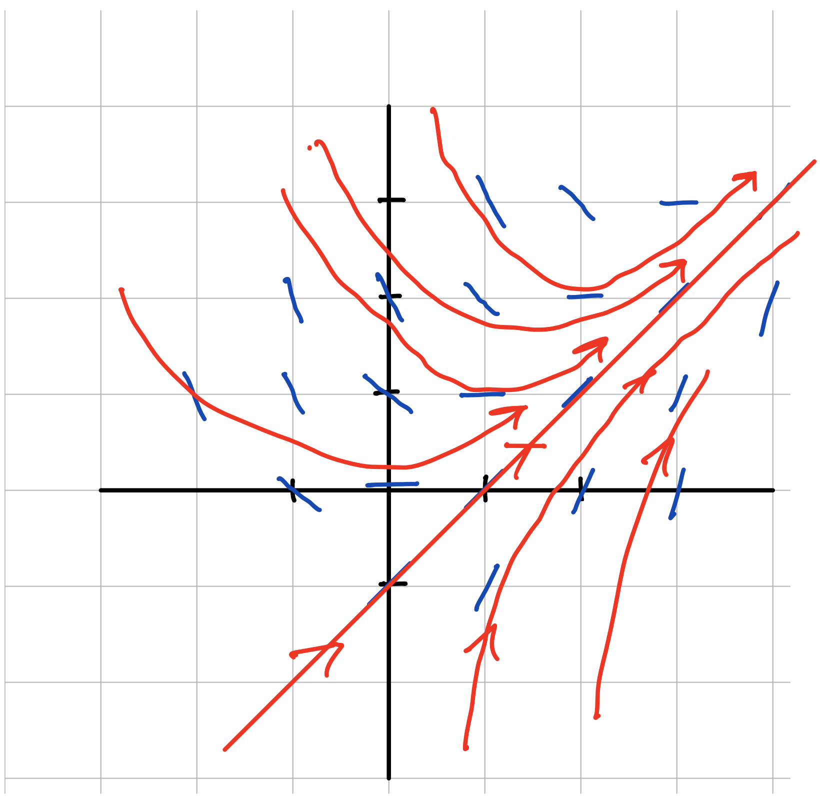


FIGURE 1.3.2(d) Slope field and solution curves for $y' = -3y$.

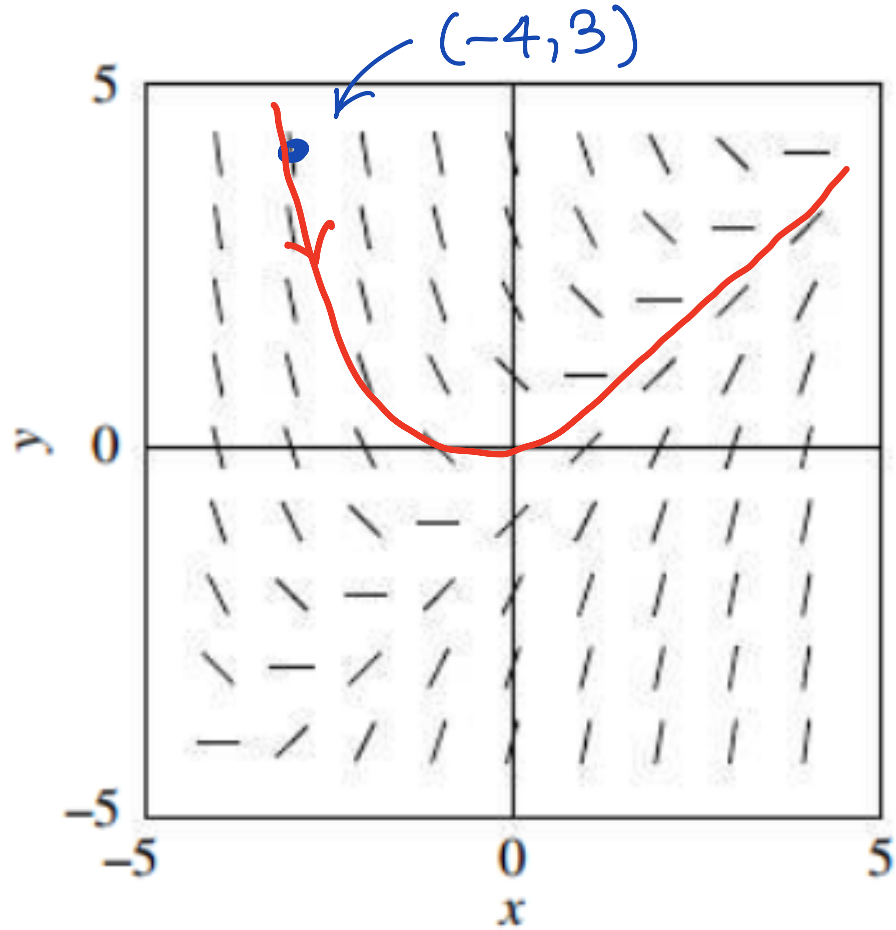
Ex: Sketch a slope field for $y' = x - y$ and some solution curves:

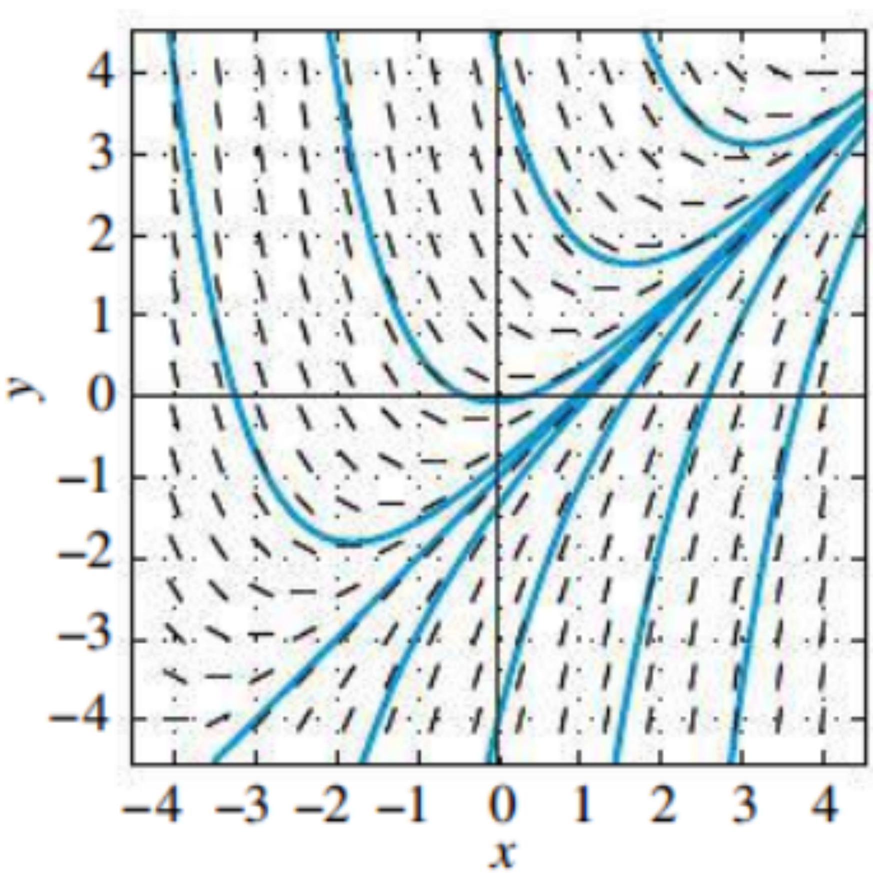


☆ A solution $y(x)$ having initial value $y(a) = b$ means sol. curve must pass through (a,b)

Solution of $y' = x - y$ having $y(-4) = 3$

(graph of part. sol. for IVP $\begin{cases} y' = x - y \\ y(-4) = 3 \end{cases}$)

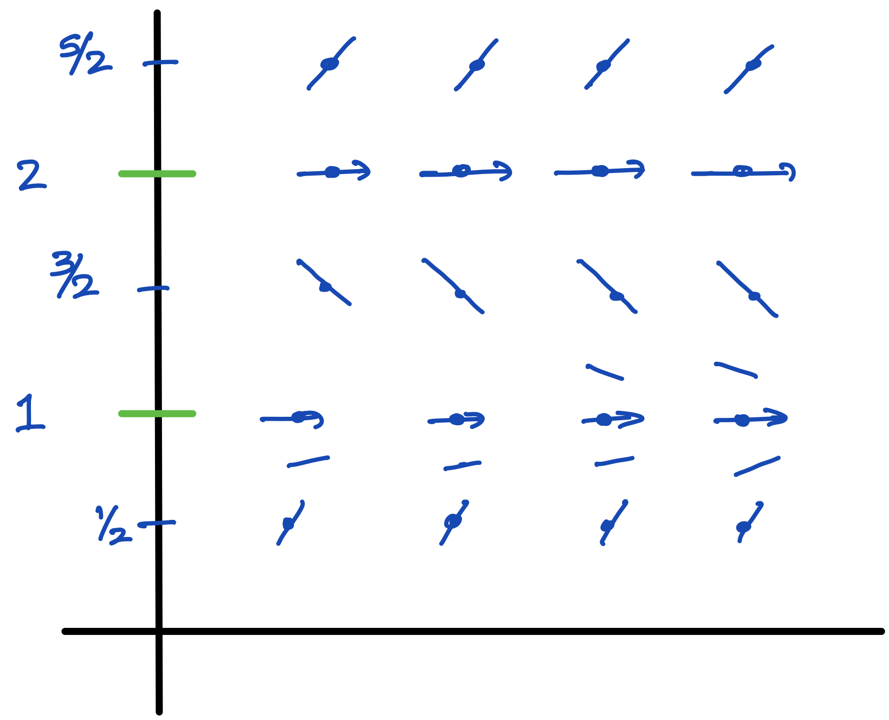




Ex what might a slope field for

$$y' = (y-1)(y-2)$$

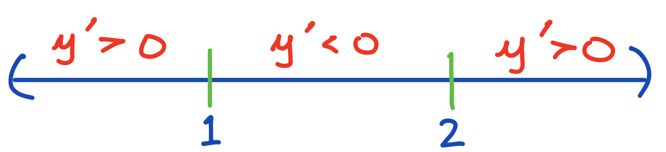
look like in the region $x \geq 0$?
How about some solution curves?



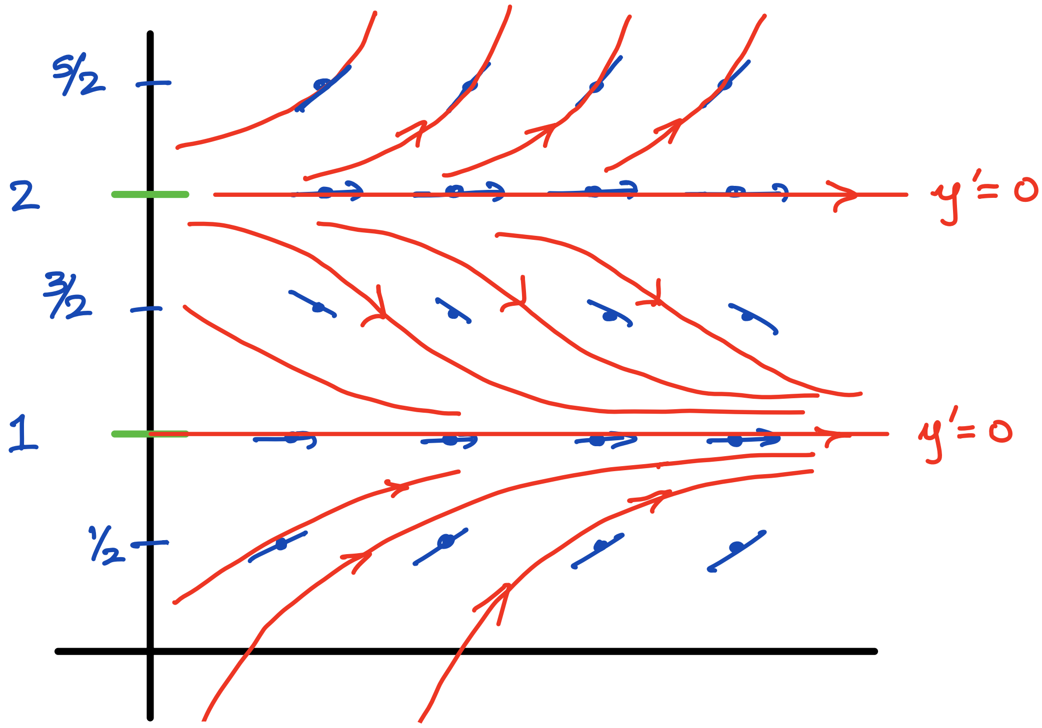
Recall ideas from calculus 1:

$$y' = (y-1)(y-2) \Rightarrow y' = 0 \text{ when } y = 1 \text{ or } 2.$$

- Also, y' doesn't depend on x.



- If $y < 1$ (and < 2),
  then $(y-1)(y-2) = (-)(-) = +$
- If $1 < y < 2$, then $y' = (+)(-) = -$
- If $y > 2$, then $y' = (+)(+) = +$



5/2
2
3/2
1
1/2
y' = 0
y' = 0

Among the following choices, which ODE has a slope field similar to the one to the right?

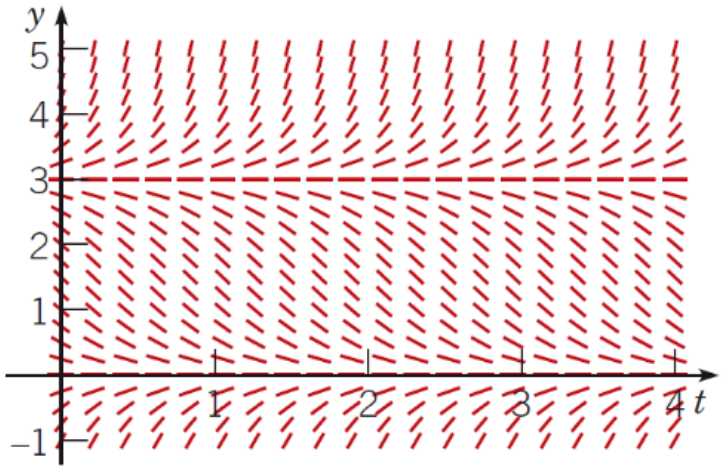
A. $y' = y(3-y)$

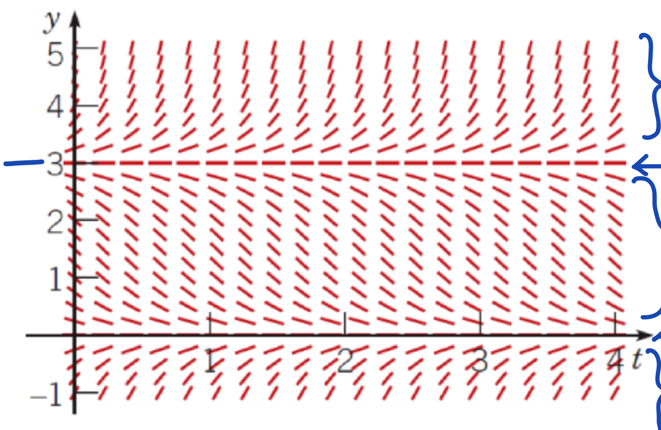
B. $y' = y-3$

C. $y' = y(3+y)$

D. $y' = y(y-3)$

E. $y' = y+3$





increasing ($y'>0$) when $y>3$

flat ($y'=0$) when $y=3$

decreasing when $0<y<3$

$y'=0$ when $y=0$

$y'>0$ when $y<0$

Thus our description of the function is

y': $+$ | $-$ | $+$

y: (—— 0 —— 3 ——) .

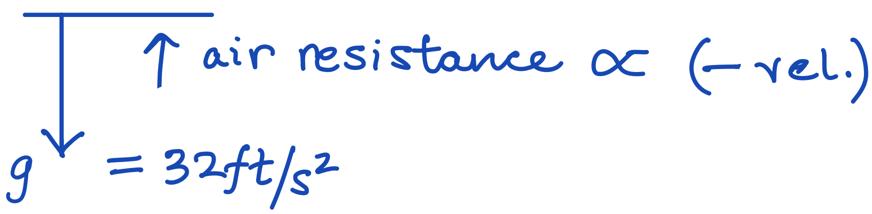
Only choices A and D have $y'=0$ when $y=0$ and $y=3$.

Choice A: $y'=y(3-y)$
$\begin{cases} \text{if } y<0, & y'=(-)(+)=- \quad \times \\ 0<y<3, & y'=(+)(+)=- \quad \times \\ y>3, & y'=(+)(-)=+ \end{cases}$

Choice D $y'=y(y-3)$ $\rightarrow$
$\begin{cases} y<0 \Rightarrow & y'=(-)(-)=+ \checkmark \\ 0<y<3 \Rightarrow & y'=(+)(-)=- \checkmark \\ y>3 \Rightarrow & y'=(+)(+)=+ \checkmark \end{cases}$

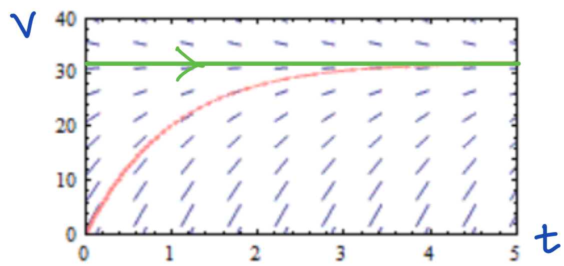
You bail out of a helicopter and pull the ripcord of your parachute. Now the air resistance proportionality constant is k = 1.6, so your downward velocity satisfies the initial value problem below, where v is measured in ft/s and t in seconds. In order to investigate your chances of survival, construct a slope field for this differential equation and sketch the appropriate solution curve. What will your limiting velocity be? Will a strategically located haystack do any good? How long will it take you to reach 95% of your limiting velocity.

$$\frac{dv}{dt} = 32 - 1.6v, \quad v(0) = 0$$

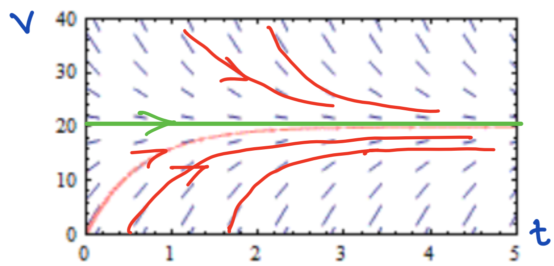


Construct a slope field for this differential equation and sketch the appropriate solution curve. Choose the correct graph below.

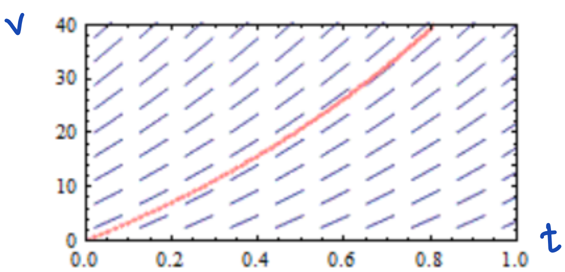
○ **A.**



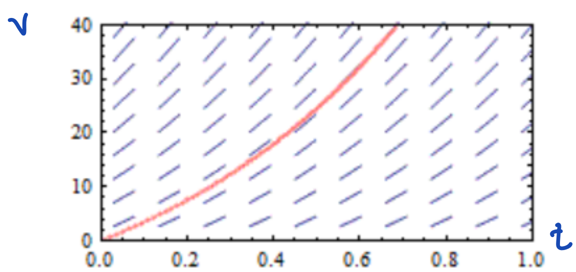
○ **B.**



○ **C.**



○ **D.**



$a(t) = g - kv$ ("down" is "+")

$v'(t) = 32 - 1.6v = 32 - \frac{16}{10}v = 16\left(2 - \frac{1}{10}v\right) = \frac{16}{10}(20 - v)$

$\frac{dv}{dt} = \frac{8}{5}(20 - v)$ (careful that this <u>isn't</u> $\frac{dv}{dt} = f(t)$)

$$\begin{cases} v < 20 \Rightarrow v' > 0 & \text{(speed inc.)} \\ v = 20 \Rightarrow v' = 0 & \text{(no change)} \\ v > 0 \Rightarrow v' < 0 & \end{cases}$$

Thus, $v(t)$ tends toward $v = 20$ (<u>terminal velocity</u>)

This is 20 ft/sec ≈ 13.6 mph … you'll live